"I know what we're going to do today."

Agenda:

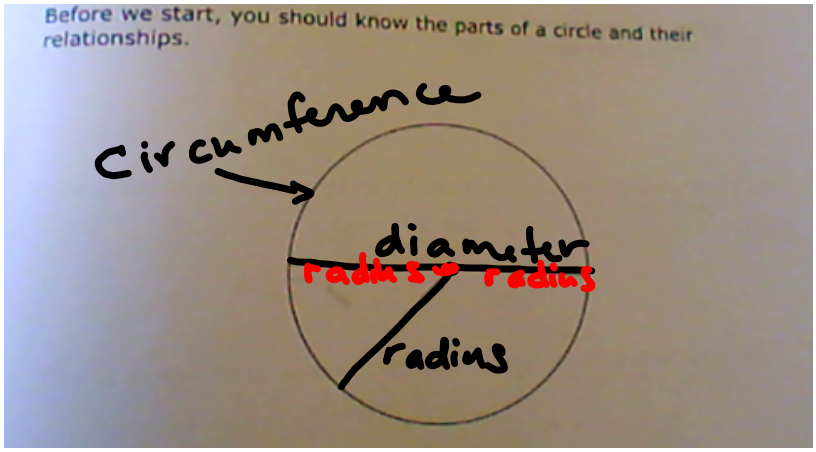
1) Warm up-

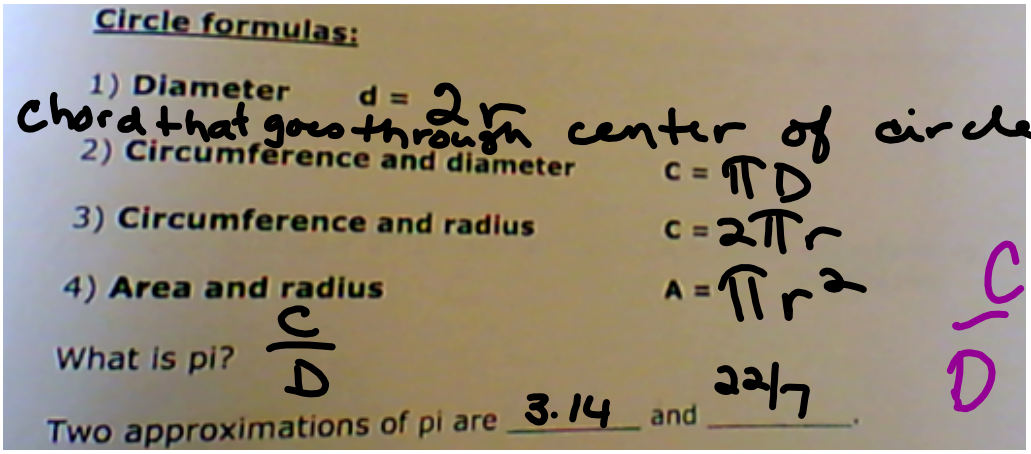
2) Unit 4 Geometry
Circumference and Area of a Circle

3) Homework: pages 83-85 (1-11)

What you'll learn in this lesson:

- How is the circumference of a circle calculated?
- How is the area of a circle calculated?
- Given the circumference of a circle, how is the radius calculated?
- Given the area of a circle, how is the radius calculated?


Before we start, you should know the parts of a circle and their relationships.
Circumference
diameter
radius
radius
radius



$\frac{C}{D} = \frac{\pi D}{D}$

$\frac{C}{D} = \pi$

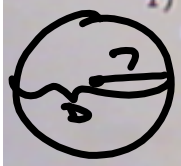
$C = \pi D$

$\frac{C}{D} = \pi$

Examples:

$r = 7$, $D =$

1) The radius of a circle is 7 cm, what is the circumference of the circle?



$C = \pi D$

$C = \pi \cdot 14$

$C = 14\pi$ cm

$C = 2\pi r$

$C = 2 \cdot \pi \cdot 7$

$C = 14\pi$ cm

2) The diameter of a circle is 10 inches, what is the area of the circle to the nearest tenth?

round

$D = 10$ in

$r = 5$ in

$A = \pi r^2$

$A = \pi 5^2$

$A = \pi \cdot 25$ (exact)

$A \approx 78.5 in^2$

3) What is the radius of a circle if the circumference of the circle is 10π cm?

$C = 2\pi r$

$\frac{10\pi}{2\pi} = \frac{2\pi r}{2\pi}$

$5 = r$

4) What is the diameter of a circle if the area of the circle is 49π cm^2?

$D = ?$

$A = \pi r^2$

$\frac{49\pi}{\pi} = \frac{\pi r^2}{\pi}$

$\sqrt{49} = \sqrt{r^2}$

$7 = r$

$14 cm = D$

5) What is the radius of a circle if the circumference is 62.8 meters?

$C = 2\pi r$

$\frac{62.8}{2\pi} = \frac{2\pi r}{2\pi}$

$10 = r$

6) What is the radius of a circle if the area is 78.5 square centimeters?

$A = \pi r^2$

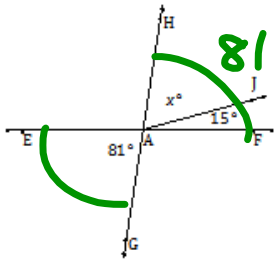
$\frac{78.5}{\pi} = \frac{\pi r^2}{\pi}$

$\sqrt{25} = \sqrt{r^2}$

$5\text{ cm} = r$

5) Find the measure of $\angle HAJ$.

81
-15



$5x + 90 + 225 = 360$

$5x + 315 = 360$

$5x = 45$

$x = 9$

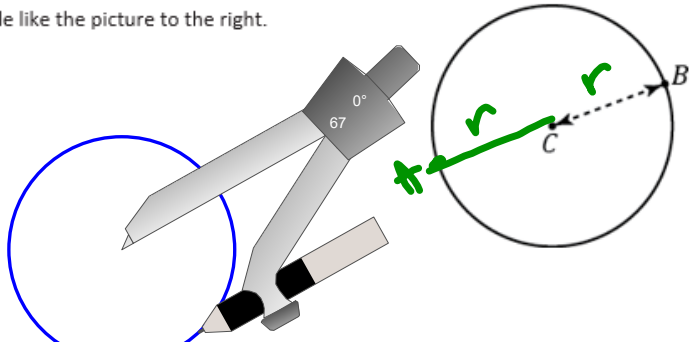
6) The sum of the measures of two adjacent angles is $72°$. The ratio of the smaller angle to the larger angle is $1:3$. Find the measures of each angle.

Lesson 4: The Most Famous Ratio of All

Classwork

Opening Exercise

a. Using a compass, draw a circle like the picture to the right.



C is the *center* of the circle.
The distance between *C* and *B* is the *radius* of the circle.

C is the *center* of the circle.

The distance between C and B is the *radius* of the circle.

b. Write your own definition for the term circle.

all points equidistant from a center point

c. Extend segment CB to a segment AB in part (a), where A is also a point on the circle.

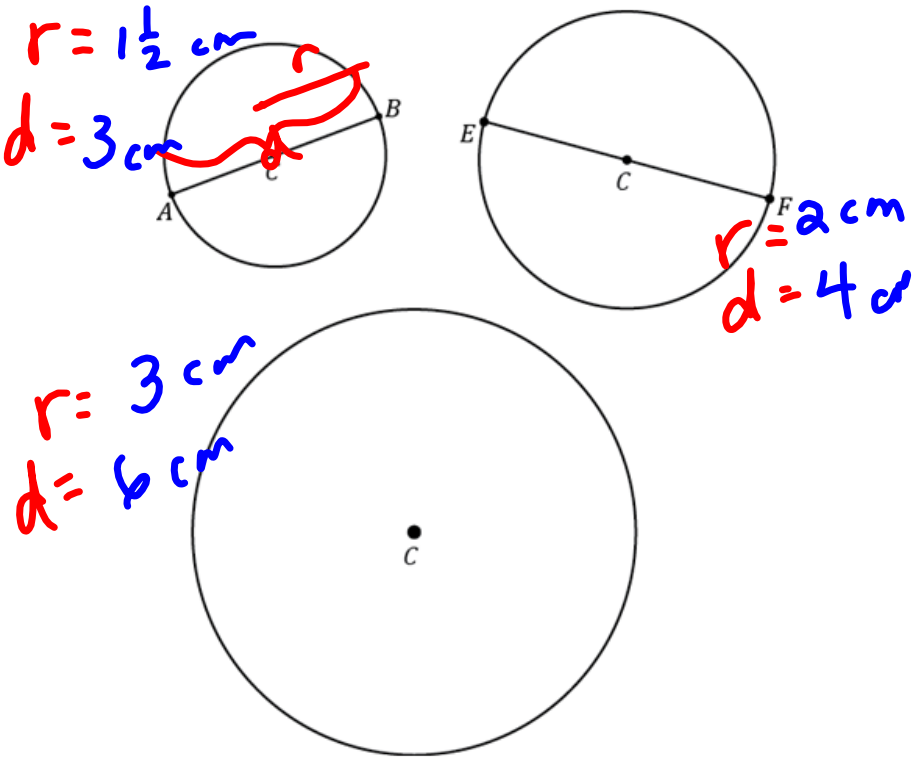
The length of the segment AB is called the *diameter of the circle.*

Diameter

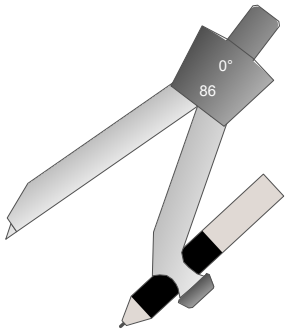
d. The diameter is twice as long as radius.

radius (pl) radii

e. Measure the radius and diameter of each circle. The center of each circle is labeled C .



f. Draw a circle of radius **6** cm.

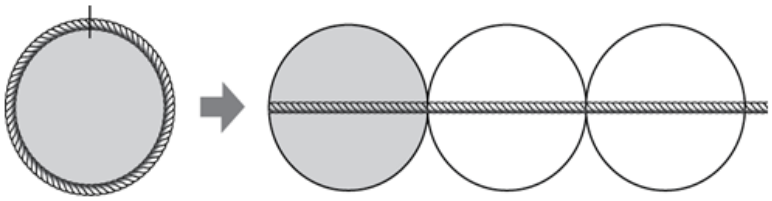


Example 1

The ratio of the circumference to its diameter is always the same for any circle. The value of this ratio,

$$\frac{Circumference}{Diameter}$$

is called the number *pi* and is represented by the symbol π .



Since the circumference is a little greater than 3 times the diameter, π is a number that is a little greater than 3.

State: Use the symbol π to represent this special number. Pi is a non-terminating, non-repeating decimal and mathematicians use the symbol π or approximate representations as more convenient ways to represent pi.

- $\pi \approx 3.14$ or $\frac{22}{7}$.
- The ratios of circumference : diameter and $\pi : 1$ are equal.
- Circumference of a Circle $= \pi \times$ Diameter.

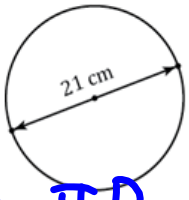
$\pi = \frac{C}{D}$

Cherry Pie is delicious

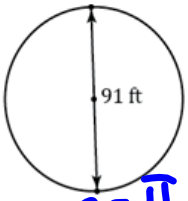
Exercise 2

$A = \pi r^2$ $C = \pi D$

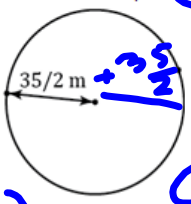
a. The following circles are not drawn to scale. Find the circumference of each circle. (Use $\frac{22}{7}$ as an approximation for π .)



$C = \pi D$
$C = \frac{22}{7}(21)3$
$C = 66$ cm



$C = \pi D$
$C = \frac{22}{7}(91)$
$C = 286$ cm



$\frac{35}{2} = \frac{22}{2} = 35$
$C = \pi D$
$C = \frac{22}{7}(35)$
$C = 110$ m

b. The radius of a paper plate is 11.7 cm. Find the circumference to the nearest tenth. (Use 3.14 as an approximation for π .)

$r = 11.7$
$D = 11.7 \times 2 = 23.4$

$C = \pi D$
$C = 3.14(23.4) \approx 73.5$ cm
73.476

c. The radius of a paper plate is 11.7 cm. Find the circumference to the nearest hundredth. (Use the π button on your calculator as an approximation for π .)

73.476

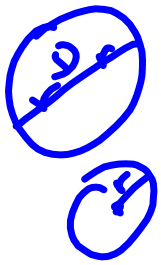
≈ 73.48 cm

d. A circle has a radius of r cm and a circumference of C cm. Write a formula that expresses the value of C in terms of r and π

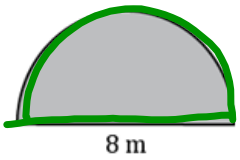
$C = \pi D$

$C = \pi(2r)$

$C = 2\pi r$



e. The figure below is in the shape of a semicirde. A semicircle is an arc that is "half" of a circle. Find the perimeter of the shape. (Use 3.14 for π .)



$\frac{\pi D}{2} + 8$

$\frac{\pi \cdot 8}{2} + 8$

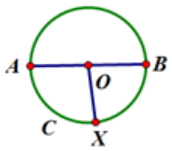
Relevant Vocabulary

Circle: Given a point C in the plane and a number $r > 0$, the *circle with center* C *and radius* r is the set of all points in the plane that are distance r from the point C.

Radius of a circle: The *radius* is the length of any segment whose endpoints are the center of a circle and a point that lies on the circle.

Diameter of a circle: The *diameter of a circle* is the length of any segment that passes through the center of a circle whose endpoints lie on the circle. If r is the *radius* of a circle, then the diameter is $2r$.

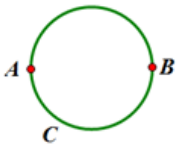
The word *diameter* can also mean the segment itself. Context determines how the term is being used: "the diameter" usually refers to the length of the segment, while "a diameter" usually refers to a segment. Similarly, "a radius" can refer to a segment from the center of a circle to a point on the circle.



Circle C

Radii:
$\overline{OA}, \overline{OB}, \overline{OX}$

Circumference

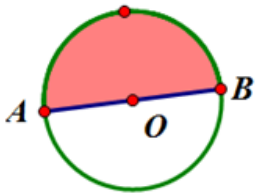


Circumference: The circumference of a circle is the distance around a circle.

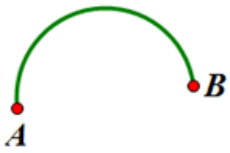
Pi: The number *pi*, denoted by π, is the value of the ratio given by the circumference to the diameter, that is $\pi = \frac{circumference}{diameter}$. The most commonly used approximations for π is 3.14 or $\frac{22}{7}$.

Semicircle: Let C be a circle with center O, and let A and B be the endpoints of a diameter. A *semicircle* is the set containing A, B, and all points that lie in a given half-plane determined by AB (diameter) that lie on circle C.

Half-plane

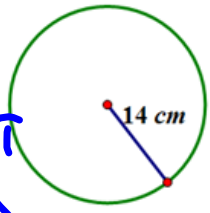


Semi-circle



Problem Set

1. Find the circumference.



a. Give an exact answer in terms of π .

$C = \pi D$

$C = \pi \cdot 28$ or $C = 28\pi$

b. Use $\pi \approx \frac{22}{7}$ and express your answer as a fraction in lowest terms.

$C = \cancel{28}^{4}\left(\frac{22}{\cancel{7}_{1}}\right) = 88$ cm

c. Use *the* π button on your calculator and express your answer to the nearest hundredth.

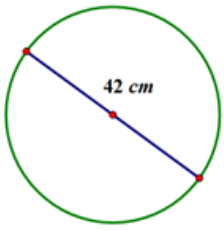
$C = 28 \cdot \pi \approx 87.96$ cm

2. Find the circumference.

a. Give an exact answer in terms of π .

$C = \pi D$

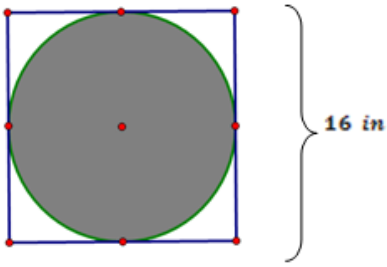
$C = \pi \cdot 42$



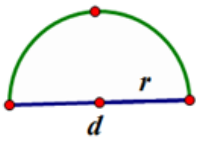
b. Use $\pi \approx \frac{22}{7}$ and express your answer as a fraction in lowest terms.

$C = \frac{22}{7} \cdot \frac{42}{1} = 132 \text{ cm}$

3. The figure shows a circle within a square. Find the circumference of the circle. Let $\pi \approx 3.14$.



4. Consider the diagram of a semicircle shown.

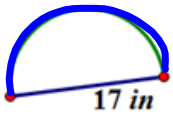


a. Explain in words how to determine the perimeter of a semicircle.

b. Using "d" to represent the diameter of the circle, write an algebraic expression that will result in the perimeter of a semicircle.

c. Write another algebraic expression to represent the perimeter of a semicircle using r to represent the radius of a semicircle.

5. Find the perimeter of the semicircle. Let $\pi \approx 3.14$.

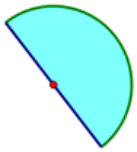


$\frac{1}{2}\pi D$

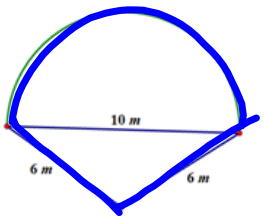
$\frac{1}{2}(17)(3.14)$

6. Ken's landscape gardening business makes odd shaped lawns which include semicircles. Find the length of the edging material needed to border the two lawn designs. Use 3.14 for π .

a. The radius of this flower bed is 2.5 m.



b. The diameter of the semicircular section is 10 m, and the lengths of the sides of the two sides are 6 m.



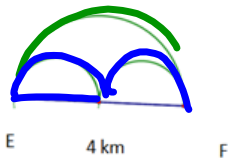
$C = \frac{1}{2}\pi D = \frac{1}{2}\pi \cdot 10 = 5\pi$

≈ 15.7

$+ \quad 12$

27.7 m

7. Mary and Margaret are looking at a map of a running path in a local park. Which is the shorter path from E to F: along the two semicircles or along the larger semicircle? If one path is shorter, how much shorter is it?



$\frac{1}{2}\pi D$

$\frac{1}{2}\pi \times 2 = \pi$

π

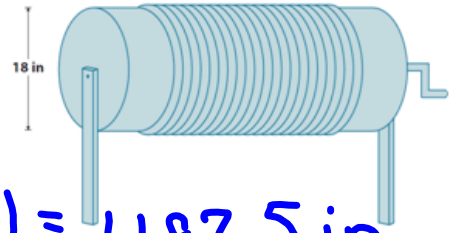
3π

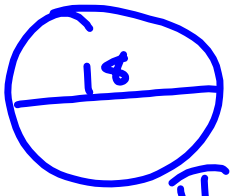
$\frac{1}{2}\pi D$

$\frac{1}{2}\cdot\pi\cdot 4 = 2\pi$

$\pi D = \pi \cdot 2$

8. Alex the electrician needs 34 yards of electrical wire to complete a job. He has a coil of wiring in his workshop. The coiled up wire is 18 inches in diameter and is made up of 21 circles of wire. Will this coil be enough to complete the job?





$C = \pi \cdot 18$

$C \approx 56.5$ in

$56.5 \times 21 = 1187.5$ in

$\frac{1187.5}{36} \approx 33$ yds

Lesson 5: The Area and Circumference of a Circle

Classwork

Opening Exercise

Solve the problem below individually. Explain your solution.

1. Find the radius of the following circle if the circumference is 37.68 inches. Use $\pi \approx 3.14$.

$$C = \pi D$$

$$\frac{37.68}{3.14} = \frac{3.14}{3.14} D$$

$$12 = D$$

$$6 = r$$

Classwork

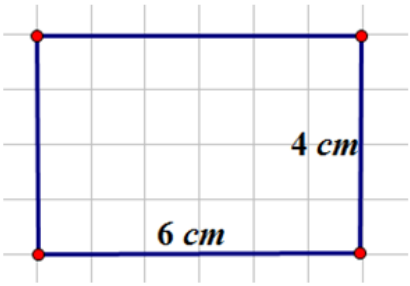
Opening Exercise

Solve the problem below individually. Explain your solution.

1. Find the radius of the following circle if the circumference is 37.68 inches. Use $\pi \approx 3.14$.

2. Determine the area of the rectangle below. Name two ways that can be used to find the area of the rectangle.

$a = 24 cm^2$



3. Find the length of a rectangle if the area is 27 cm^2 and the width is 3 cm.

$\ell = ?$

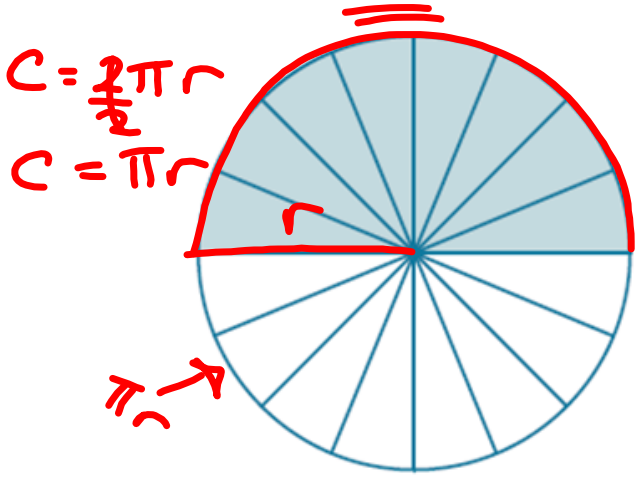
$A = \ell \cdot w$

$\frac{27}{3} = \frac{\ell \cdot 3}{3}$

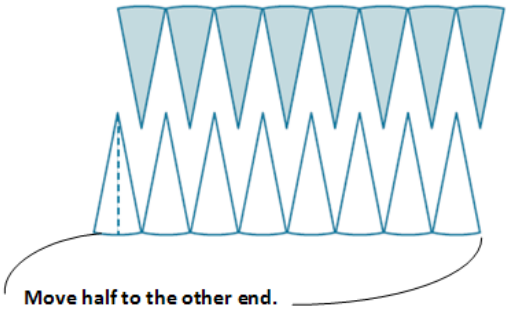
$9 = \ell$

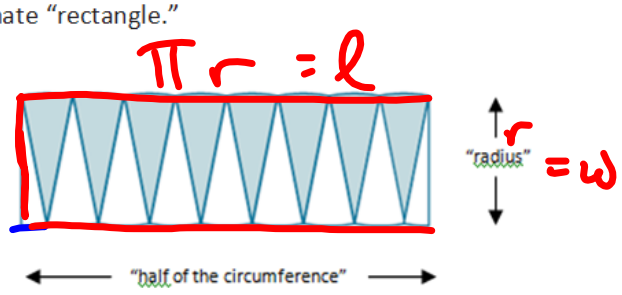
Discussion

To find the formula for the area of a circle, cut a circle into 16 equal pieces:



Arrange the triangular wedges by alternating the "triangle" directions and sliding them together to make a "parallelogram." Cut the triangle on the left side in half on the given line, and slide the outside half of the triangle to t other end of the parallelogram in order to create an approximate "rectangle."



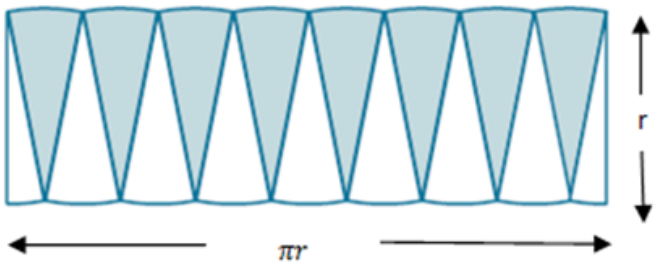


$a = l \cdot w$

$a = \pi r \cdot r$

$a = \pi r^2$

The circumference is $2\pi r$, where the radius is "r." Therefore, half of the circumference is πr .



What is the area of the "rectangle" using the side lengths above?

$\text{area} = \pi r \cdot r = \pi r^2$

$C = \pi D$

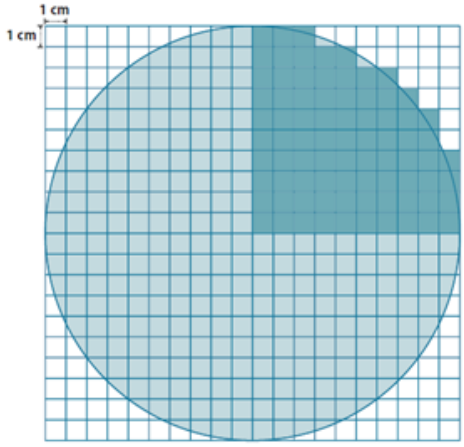
$A = \pi r^2$

Are the areas of the rectangle and the circle the same?

Yes, since we just rearranged pieces of the circle to make the "rectangle," the area of the "rectangle" and the area of the circle are approximately equal. Note that the more sections we cut the circle into, the closer the approximation. If the area of the rectangular shape and the circle are the same, what is the area of the circle?

Example 1

Use the shaded square centimeter units to approximate the area of the circle.



What is the radius of the circle?

What would be a quicker method for determining the area of the circle other than counting all of the squares in the entire circle?

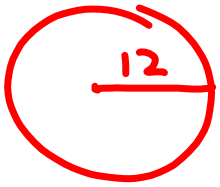
Using the diagram, how many squares did Michael use to cover one-fourth of the circle?

What is the area of the entire circle?

Example 2

A sprinkler rotates in a circular pattern and sprays water over a distance of 12 feet. What is the area of the circular region covered by the sprinkler? Express your answer to the nearest square foot.

Draw a diagram to assist you in solving the problem. What does the distance of 12 feet represent in this problem?



$a = \pi r^2$
$a = \pi (12^2)$
$a \approx 452 \text{ ft}^2$

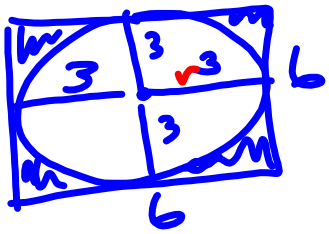
What information is needed to solve the problem?

1. equation $a = \pi r^2$
2. exponent
3. mult. by π

Example 3

Suzanne is making a circular table out of a square piece of wood. The radius of the circle that she is cutting is 3 feet. How much waste will she have for this project? Express your answer to the nearest square foot.

Draw a diagram to assist you in solving the problem. What does the distance of 3 feet represent in this problem?



square - circle
$6 \cdot 6 - \pi 3^2$
$36 - 28$
$8\ ft^2$

What information is needed to solve the problem?

① area of square
② area of circle

Does your solution answer the problem as stated?

① square $6 \times 6 = 36\ ft^2$

② circle $a = \pi r^2$
$a = \pi \cdot 3^2$
$a = \pi \cdot 9 \approx 28\ ft^2$

③ waste $36 - 28 = 8\ ft^2$

Exercises

4. A circle has a radius of 2 cm.

   a. Find the exact area of the circular region.

$a = \pi r^2$

$a = \pi 2^2 = 4\pi$

   b. Find the approximate area using 3.14 to approximate π.

$a \approx 4 \times 3.14 \approx 12.56$

5. A circle has a radius of 7 cm.

a. Find the exact area of the circular region.

$a = \pi \cdot 7^2 = 49\pi$

b. Find the approximate area using $\frac{22}{7}$ to approximate π.

$a = 49 \cdot \frac{22}{7} \approx 154$

c. What is the circumference of the circle?

$r = 7$

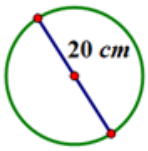
$D = 14$

$C = \pi D$

$C = \pi \cdot 14$

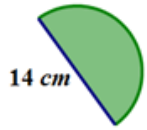
14π

6. Joan determined that the area of the circle below is $400\pi\ cm^2$. Melinda says that Joan's solution is incorrect; she believes that the area is $100\pi\ cm^2$. Who is correct and why?

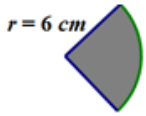


Example 4

Find the area of the following semicircle.



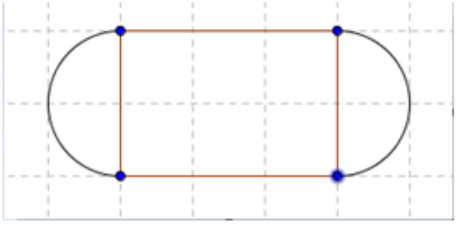
What is the area of the quarter circle?



Example 5

Marjorie is designing a new set of placemats for her dining room table. She sketched a drawing of the placement on graph paper. The diagram represents the area of the placemat consisting of a rectangle and two semicircles at either end. Each square on the grid measures 4 inches in length.

Find the area of the entire placemat. Explain your thinking regarding the solution to this problem.



If Marjorie wants to make six placemats, how many square inches of fabric will she need?

Marjorie decides that she wants to sew on a contrasting band of material around the edge of the placemats. How much binding material will Marjorie need?

Example 6

The circumference of a circle is 24π cm. What is the exact area of the circle?

Draw a diagram to assist you in solving the problem.

What information is needed to solve the problem?

Next, find the area.

Exercises

7. Find the area of a circle with a diameter of 42 cm. Use $\pi \approx \frac{22}{7}$.

8. The circumference of a circle is 9π cm.
   a. What is the diameter?

   b. What is the radius?

   c. What is the area?

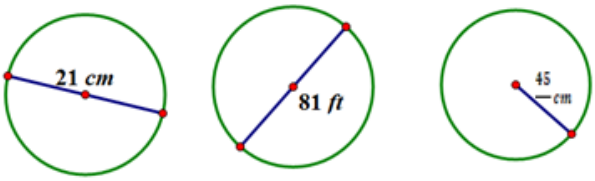
Relevant Vocabulary

Circular Region (or Disk): Given a point C in the plane and a number $r > 0$, the *circular region (or disk) with center* C *and radius* r is the set of all points in the plane whose distance from the point C is less than or equal to r.

The boundary of a disk is a circle. The "area of a circle" refers to the area of the disk defined by the circle.

Problem Set

1. The following circles are not drawn to scale. Find the area of each circle. (Use $\frac{22}{7}$ as an approximation for π.)



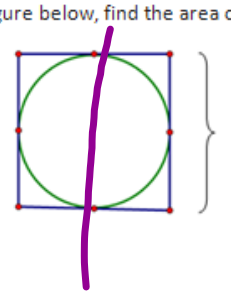
2. A circle has a diameter of 20 inches.

   a. Find the exact area and find an approximate area using ≈ 3.14 .

   b. What is the circumference of the circle using $\pi \approx 3.14$?

$100\pi \approx 314 in^2$

3. Using the figure below, find the area of the circle.



$D = 10$

$r = 5$

$a = 25\pi$ $a = \pi r^2$

4. A path bounds a circular lawn at a park. If the path is 132 ft. around, approximate the amount of area of the lawn inside the circular path. Use $\pi \approx \frac{22}{7}$.

$C = \pi D$

$\frac{132}{\frac{22}{7}} = \frac{22}{7} D$ $\quad 132\left(\frac{7}{22}\right)$

$42 = D$

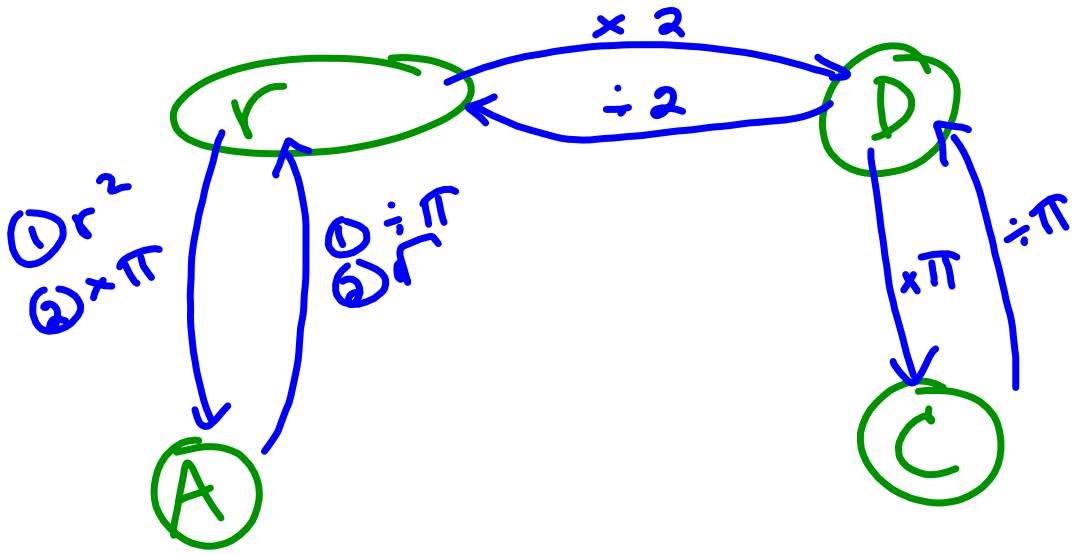
$42 \div 2 = 21 = r$

$A = \pi r^2$

$A = \frac{22}{7}(21)^2$

$A = \frac{22}{7}(441)$

$A = 1386 \text{ ft}^2$


x 2
÷ 2
r
D
A
C
① r^2
② $\times \pi$
① ÷ π
② $\sqrt{\ }$
$\times \pi$
$\div \pi$

5. Find the ratio of the area of two circles with radii 3 cm and 4 cm.

9π | 16π $\frac{9\pi}{16\pi}$

6. If one circle has a diameter of $10\ cm$ and a second circle has a diameter of $20\ cm$, what is the ratio between the areas of the circular regions?

25π 100π $\frac{25\pi}{100\pi} = \frac{1}{4}$

7. If the diameter of a circle is double the diameter of a second circle, what is the ratio of area of the first circle to the area of the second?

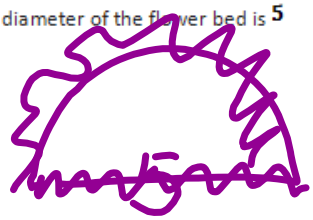
8. Mark created a flowerbed that is semicircular in shape, as shown in the image. The diameter of the flower bed is 5 m.

a. What is the perimeter of the flower bed? (Approximate π to be 3.14 .)

$C = \frac{\pi D}{2} + 5$

$C = \frac{3.14 \cdot 5}{2} + 5$

$C = 12.85$ m



b. What is the area of the flowerbed? (Approximate π to be 3.14 .)

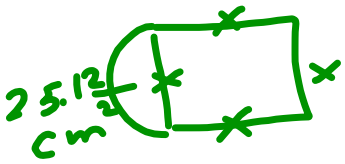
$A = \frac{\pi r^2}{2}$

$A = \frac{3.14(2.5^2)}{2}$

$A \approx 9.8 \text{ m}^2$

9. A landscape designer wants to include a semicircular patio at the end of a square sandbox. She knows that the area of the semicircular patio is $25.12\ cm^2$.

a. Draw a picture to represent this situation.



b. What is the length of the side of the square?

diameter = 2(4)
= 8

$A = \frac{\pi r^2}{2}$

(2) $25.12 = \frac{\pi r^2}{2}$ (2)

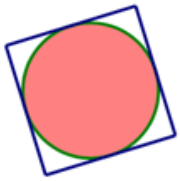
$50.24 = \pi r^2$

$\frac{50.24}{\pi} = \frac{\pi r^2}{\pi}$

$\sqrt{16} = \sqrt{r^2}$

$4 = r$

10. The figure below shows a circle inside of a square. If the radius of the circle is 8 cm, find the following and explain your solution.

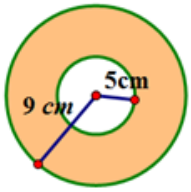


a. The circumference of the circle.

b. The area of the circle.

c. The area of the square.

11. A graphic artist is designing a company logo with two concentric circles (two circles that share the same center but have different length radii). The artist needs to know the area of the shaded band between the two concentric circles. Explain to the artist how he would go about finding the area of the shaded region.



1. Use the following expression below.

$4x - 3(x - 2y) + \frac{1}{2}(6x - 8y)$

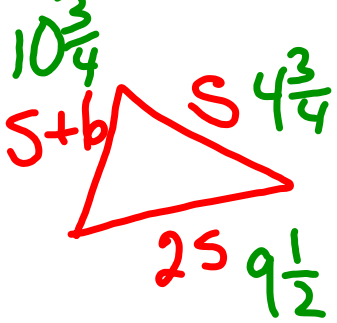
Write an equivalent expression in standard form and collect like terms.

$4x - 3x + 6y + 3x - 4y$

$4x + 2y$

2. Use the following information to solve the problems below.

   a. The largest side of a triangle is six more units than the smallest side. The third side is twice the smallest side. If the perimeter of the triangle is 25 units, write and solve an equation to find the lengths of all three sides of the triangle.



$s+6+s+2s=25$

$4s+6=25$

$-6 \quad -6$

$\frac{4s}{4}=\frac{19}{4}$

$s=4.75$ $4\frac{3}{4}$

b. The length of a rectangle is $(x+3)$ inches long, and the width is $3\frac{2}{5}$ inches. If the area is $15\frac{3}{10}$ square inches, write and solve an equation to find the length of the rectangle.

$x+3$

$3\frac{2}{5}$ | $15\frac{3}{10}$

$3\frac{2}{5}(x+3)=15\frac{3}{10}$

$3\frac{2}{5}x+10\frac{1}{5}=15\frac{3}{10}$

$-10\frac{1}{5}\quad -10\frac{1}{5}=\frac{2}{10}$

$3\frac{2}{5}x=5\frac{1}{10}$

$3\times 3=9$

$\frac{2}{5}\times 3=\frac{6}{5}=1\frac{1}{5}$

$3\frac{2}{5}$

$x=1\frac{1}{2}$

$5\frac{1}{10}\div 3\frac{2}{5}$

$\frac{51}{10}\div\frac{17}{5}$

$\frac{\cancel{51}^{3}}{\cancel{10}_{2}}\times\frac{\cancel{5}^{1}}{\cancel{17}_{1}}=\frac{3}{2}$

3. A picture $10\frac{1}{4}$ feet long is to be centered on a wall that is $14\frac{1}{2}$ feet long. How much space is there from the edge of the wall to the picture?

4. In August, Cory begins school shopping for his triplet daughters.

   a. One day, he bought 10 pairs of socks for $\$2.50$ each and 3 pairs of shoes for d dollars each. He spent a total of $\$135.97$. Write and solve an equation to find the cost of one pair of shoes.

   b. The following day Cory returned to the store to purchase some more socks. He had $\$40$ to spend. When he arrived at the store, the shoes were on sale for $\frac{1}{3}$ off. What is the greatest amount of pairs of socks Cory can purchase if he purchased another pair of shoes in addition to the socks?

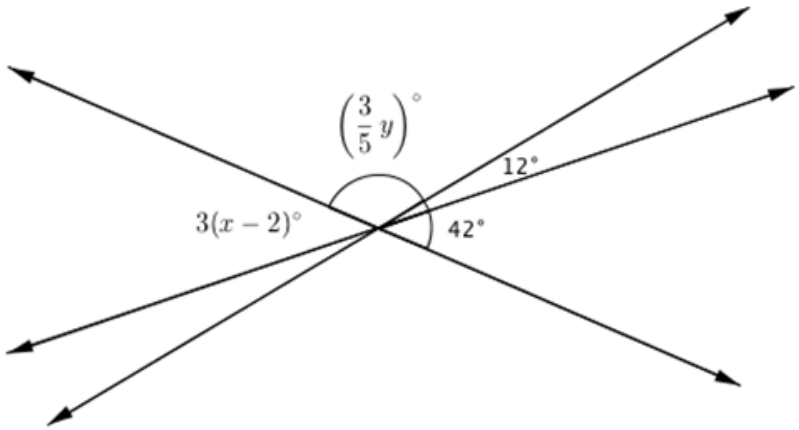
5. Jenny invited Gianna to go watch a movie with her family. The movie theater charges one rate for 3D admission and a different rate for regular admission. Jenny and Gianna decided to watch the newest movie in 3D. Jenny's mother, father, and grandfather accompanied Jenny's little brother to the regular admission movie.

   a. Write an expression for the total cost of the tickets. Define the variables.

b. The cost of the 3D ticket was double the cost of the regular admission ticket. Write an equation to represent the relationship between the two types of tickets.

c. The family purchased refreshments and spent a total of $18.50. If the total amount of money spent on tickets and refreshments were $94.50, use an equation to find the cost of one regular admission ticket.

6. The three lines shown in the diagram below intersect at the same point. The measures of some of the angles in degrees are given as $3(x-2)°$, $\left(\frac{3}{5}y\right)°$, $12°$, $42°$.



a. Write and solve an equation that can be used to find the value of x.

b. Write and solve an equation that can be used to find the value of y .